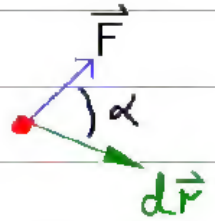
Meriam 3/6 : Work & Energy

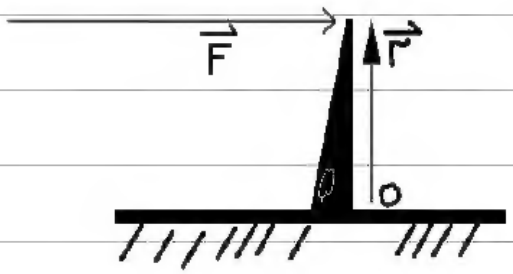
Definition: Work done by a force $\vec{F}$ on a particle over a small displacement $d\vec{r}$:

$$dU = \vec{F} \cdot d\vec{r}$$
$$= |\vec{F}||d\vec{r}|\cos\alpha \quad (+/-)$$



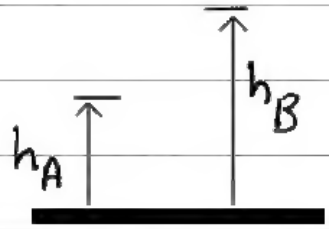
Note:

- work done by a force can be +ve or -ve
- Work is a scalar (no associated direction)
- units of work are N·m or Joules (J)
- Express explicitly which force is doing work on which object
- Do not confuse work with a torque or a moment $\vec{M}$ (vector) which also has units N·m:



$$\vec{M}_O = \vec{r} \times \vec{F}$$

$W_{gravity}$ (near surface of the earth)



$h_B > h_A$

Raise a mass m from h_A to h_B

$$U_g = \int_{h_A}^{h_B} \vec{F} \cdot d\vec{r}$$

$$U_g = \int_{h_A}^{h_B} -(F)(dr) = \int_{h_A}^{h_B} -(F)(dh)$$

$$U_g = -mg\Delta h$$ (U_g is negative, Δh is +ve)

Lower a mass m from h_B to h_A

$$U_g = \int_{h_B}^{h_A} \vec{F} \cdot d\vec{r}$$

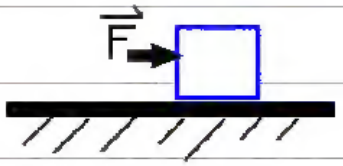
$$U_g = \int_{h_B}^{h_A} +(F)(dr) = \int_{h_B}^{h_A} +(F)(-dh)$$

$$U_g = -mg\Delta h$$ (U_g is +ve, Δh is −ve)

$U_g = -mg\Delta h$ if object is being raised or lowered

Note that only the vertical component of movement contributes towards U_g

$$U_{\text{static friction}} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = 0$$

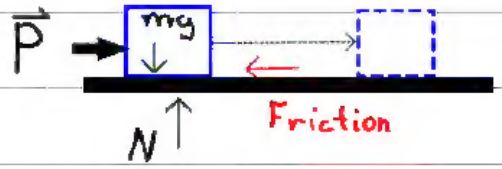


But consider the case of a box on an accelerating truck:
$\vec{F} \neq 0$; $\Delta\vec{r} \neq 0$; $U_{\text{friction}} \neq 0$

$$U_{\text{kinetic friction}} = \int_1^2 \vec{F} \cdot d\vec{r}$$

$$= -F_f \Delta s$$

$$U_{\text{kinetic friction}} = -\mu_k N \Delta s$$



Springs : $F = -kx$

where F = force exerted by the sping
x = spring extension from rest length

$$U_{\text{spring}} = \int_{x_1}^{x_2} \vec{F} \cdot d\vec{r} = \int_{x_1}^{x_2} -kx\, dx$$

$$U_{1\to 2} = -\frac{1}{2} k \left(x_2^2 - x_1^2\right)$$

→ A spring does positive work as it moves towards its relaxed length

Always check the +/- sign of your work calculations

Key idea:

$$dU = \vec{F} \cdot d\vec{r} = F_t\, ds = m a_t\, ds$$

$$\rightarrow U_{1\rightarrow 2} = \int_1^2 dU = \int_1^2 F_t\, ds = \int_1^2 m a_t\, ds$$

but $m(a_t\, ds = v\, dv)$ $\rightarrow$ $U_{1\rightarrow 2} = \int_1^2 mv\, dv$

$$\rightarrow \quad U_{1\rightarrow 2} = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2$$

Define kinetic energy $T = \frac{1}{2} m v^2$

$$\rightarrow \boxed{T_2 = T_1 + U_{1\rightarrow 2}}$$

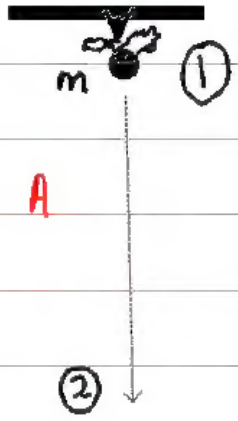
- where $U_{1\rightarrow 2}$ is the work done on the mass by forces as it moves from position 1 to position 2.
- This is an energy-balance equation. All energy sources & sinks must be accounted for.
- We now have 2 methods to find final v at end of trip

(1) $T_2 = T_1 + U_{1\rightarrow 2} \rightarrow v_2$

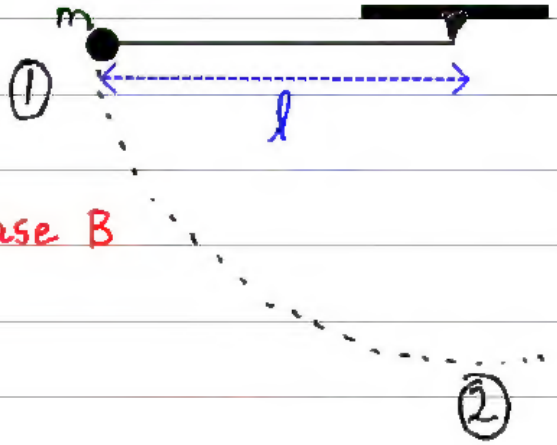
(2) Determine $\vec{a}(t) = \vec{F}(t)/m$ for entire trip

$$\rightarrow \vec{v}_2 = \vec{v}_1 + \int_1^2 \vec{a}(t)\, dt$$

Eg. Drop 2 string pendulums of length l from rest. Find V when mass m approaches point ②



case A



case B

Solution

Case A: rectilinear motion: $a = g\downarrow$

$$V_2^2 = \cancel{V_1^2}^{\,0} + (2)(a)(\Delta s)$$

$$V_2^2 = 0 + (2)(g)(l)$$

$$\boxed{V_2 = \sqrt{2gl}}$$

or $T_2 = T_1 + U_{1\to 2}$

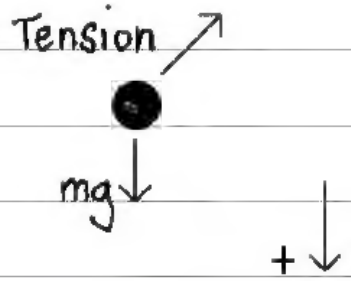
$$\tfrac{1}{2} m V_2^2 = \tfrac{1}{2} m \cancel{V_1^2}^{\,0} - mg\Delta h$$

$$\tfrac{1}{2} \cancel{m} V_2^2 = 0 - \cancel{m} g(-l)$$

$$\rightarrow \boxed{V_2 = \sqrt{2gl}}$$

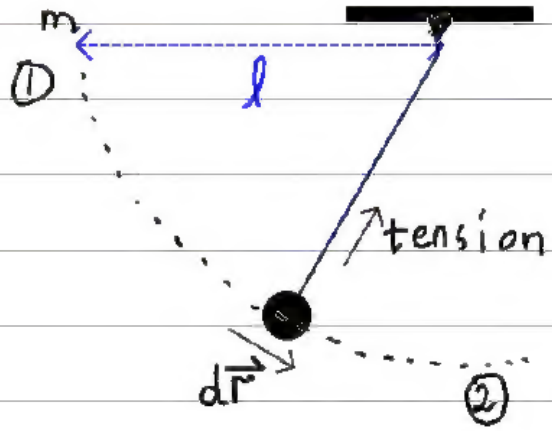
case B

$T_2 = T_1 + U_{1\to 2}$, where $dU = \vec{F}\cdot d\vec{r}$



Tension in rope is always $\perp$ to $d\vec{r}$

→ No work done by rope!!



$\frac{1}{2} m v_2^2 = \frac{1}{2} m v_1^2 - mg\Delta h$ ($v_1 = 0$)

$\frac{1}{2} m v_2^2 = 0 - mg(-l)$

→ $\boxed{v_2 = \sqrt{2gl}}$

Note that case B would be difficult to solve by $\int a\,dt$

Summary

- $T_2 = T_1 + U_{1\to 2}$ to solve for final v or distance traveled
- In some cases, $U_{1\to 2}$ can be hard to find
- Easy expressions for U_g, U_e, $U_{friction}$

Next Lecture

- Practical Applications of Work/Energy equation